

It Is All Relative: How Young Children Encode Extent

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Two experiments tested the ability of 4- and 8-year-old children to encode the extent of a target dowel and later discriminate between the target and a foil having a novel extent. By manipulating the heights of containers in which we presented the stimuli we tested whether children used the relation between the dowels and containers for encoding extent. We found that 8-year-olds encoded extent without relying on the relation between the target dowel and container but 4-year-olds only encoded the extent of the target dowel relative to the container. This early ability to encode extent relative to an aligned standard may serve as a perceptual basis for the developing ability to measure.

This article investigates how young children encode and retain information about the extent of a continuous quantity. The ability to determine extent underlies many important cognitive functions recruited in a number of everyday tasks. For example, identifying an object as one seen earlier or establishing the quantitative equivalence in length, area, or volume of different objects requires the ability to encode and represent continuous amount. Although a significant amount of research has focused on the origins of discrete numerical quantification in infancy and early childhood, much less is known about how infants and young children encode information about continuous quantities.

Older children and adults encode and retain information about extent by imposing a standard or measure such as a ruler or thumb-length on a target object. The

extent of the object is encoded as a ratio or proportion of the standard, such as 3 in. or $\frac{1}{2}$ L. This relation may be used later to compare or discriminate different objects that differ only in extent or to reconstruct the target amount. Piaget and his colleagues claimed that young children do not spontaneously impose standards on objects to measure (Piaget, Inhelder, & Szeminska, 1960). They argued that imposing a standard requires an understanding of the logic of transitive inferences, a mental operation that Piaget claimed emerges only in the school age years. Piaget therefore argued that infants and young children have a very limited capacity to encode or retain metric information about an object's spatial extent.

In contrast to Piaget's claim, several more recent studies demonstrate sensitivity to extent in infants and young children. Baillargeon (1991) found that infants as young as 4 months are sensitive to an occluded object's height. Infants habituated to a rotating drawbridge dishabituate when the drawbridge appears to pass through a portion of a solid object, indicating that the infants encoded information about the height of that object. Newcombe, Huttenlocher, and Learmonth (2000) showed that 6-month-olds remember the location of a hidden object in a long, narrow sandbox, demonstrating an early ability to represent distance. Gao, Levine, and Huttenlocher (2000) found that 6-month-old infants can discriminate between two identical containers holding different amounts of liquid, suggesting that infants were sensitive to the difference in liquid volume. A variety of studies have also demonstrated sensitivity to extent among young children (i.e., Bryant, 1974; Miller, 1989; Miller & Baillargeon, 1990). If Piaget's analysis is correct and infants and young children cannot measure, it is unclear how they encoded extent in the studies previously cited.

One possible explanation for this early sensitivity to extent is that although infants and young children do not impose standards on objects, they may use an available aspect of a stimulus display as a standard to help encode the extent of a target object. For example, in the infant studies previously cited, the target object or extent was always aligned with another object that could serve as a standard object, such as an occluding screen (Baillargeon, 1991), a sandbox (Newcombe et al., 2000), or a glass container (Gao et al., 2000). It is possible that in the absence of such an aligned object, infants and young children might be unable to encode and retain information about extent.

Huttenlocher, Duffy, and Levine (2002) explored this possibility by presenting infants and young children with objects in the presence or absence of aligned standards. Six-month-old infants were habituated to a target dowel either in isolation, beside a gray wooden stick, or inside a glass container. Once habituated to the display, infants were alternately presented with the target dowel and a novel dowel that differed only in height. Infants looked longer at the novel dowel when the target and novel dowel were presented either beside the gray stick or inside the container. However, infants did not dishabituate to the novel dowel when the dowels were presented without an accompanying standard. This finding suggests that infants used the container or the gray stick to encode the extent of the dowels.

Huttenlocher and colleagues (2002) also tested whether 2- and 4-year-old children could encode the extent of a target dowel in the absence of an aligned standard in a forced-choice discrimination task. As in the infant study, children were presented with a target stimulus, either alone or inside a container. This display was removed, and children were required to discriminate between the target dowel and a foil dowel that differed only in height. When the dowels were presented with accompanying standards, children successfully chose the target dowel in the discrimination task. However, when the dowels were presented without a standard object, 2-year-old children, like infants, were unable to discriminate between the target and the foil. In contrast, 4-year-old children, like adults, demonstrated an ability to encode the target's height in the absence of a standard.

Although the Huttenlocher and colleagues (2002) finding demonstrated that infants and young children do not encode extent in the absence of an aligned standard object, the study did not address how children use such an object when one is present. One possibility is that although infants and young children cannot impose an external or mental standard on an object like adults, young children may automatically encode the relation between the target object and the aligned standard. If this hypothesis is correct, then changing the size of a standard between object comparisons should disrupt the ability to discriminate a target object from another object that has a novel extent. Moreover, if children only encode the extent of an object as a relation between the object and an aligned standard, children should be misled into thinking that two objects that differ in extent but that have the same relation to their accompanying standards are identical in extent.

Another possibility is that children do not use the standard to encode relative information but that the presence of a second object such as a container merely highlights or accentuates the extent of a target object. In such cases, children may impose an external or mental standard on the target object like adults but only fail to do so when an object is presented in isolation. If this hypothesis is true, then the actual size of the standard or its relation to the target object should not affect the ability to encode extent.

This study tests these two possibilities by exploring how 4- and 8-year-old children use an available standard for encoding extent in two experiments that manipulate the relation between a target and an aligned standard. We chose these target age groups because Huttenlocher and colleagues (2002) found that children begin to encode extent in the absence of an aligned standard around 4 years of age and exhibit mature measurement by 8 years of age (Piaget et al., 1960).

In both experiments, we presented children with trials consisting of two phases: an initial target presentation in which children are presented with a dowel inside a container and a choice task in which the children are shown the target and a foil and must choose the original target. In two experimental conditions and a control condition we test the hypothesis that young children rely on the relation between the initial target and its container for discriminating the target from the foil in the

choice task. The control condition is designed to replicate the container condition in the initial study by Huttenlocher and colleagues (2002) using the dowel stimuli used in this investigation. Critical to this study, the containers used in the initial target presentation and choice task are identical in height in this control condition. However, in both experimental conditions, the two containers in the choice task are the same height as each other but differ in extent from the container used in the target presentation. This switch in container sizes between the two phases of the experiment changes the relative information in the display so that the target dowel in the choice task no longer has the same relation to the changed container as the target dowel had to the container used in the initial target presentation. Encoding only the initial relation between the target and container provides no information for discriminating the target from the foil in the choice task.

However, the critical manipulation in the experimental conditions concerns the relation between the changed container and the foil dowel. In the relative foil condition, the change in container size between the two phases of the experiment is designed so that the foil dowel occupies the same proportion of the new container as the target had in the original container (see Figure 1). If children only encode the relation between the target and the container in the initial phase of the trial, they should mistake this “relative foil” for the target dowel and thus the proportion of trials in which children choose the target should be significantly below chance. However, such performance does exclude the possibility that children can encode extent without relying on relative information but may be drawn to choosing the foil because it may appear more perceptually similar to the initial target or that children may have misunderstood the task demands. To eliminate this possibility we introduce the unrelated foil condition in Experiment 2. In this condition, the foil dowel does not have the same relation or extent as the target in the initial phase of the trial (see Figure 1). Because in this condition neither target nor foil have the same relation to the changed container that the target had in the initial display, children should exhibit chance performance at discriminating the target from the unrelated foil if they only encoded the relation between the initial target and container.

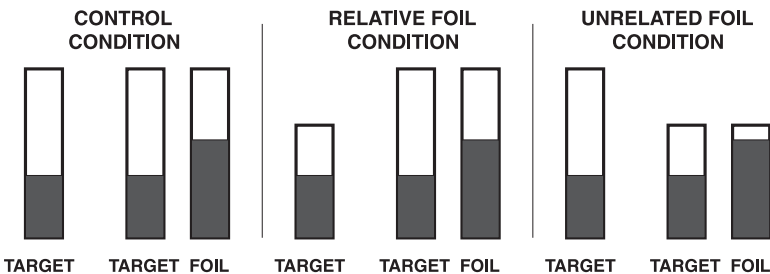


FIGURE 1 Sample schematic trials of the three conditions used in this study.

In the first experiment, we test 4-year-old children in the relative foil condition and in the control condition, although in the second experiment we test 4- and 8-year-old children in the relative foil and the unrelated foil conditions.

EXPERIMENT 1

In Experiment 1, 4-year-old children were tested in two within-subjects conditions: the control condition and the relative foil condition previously described. The control condition was included to replicate Huttenlocher and colleagues' (2002) finding with the dowel stimuli used in this study and to determine whether children's performance is above chance when relative information remains consistent between the initial target presentation and choice task. If children encode only the relation between the target and container and use that relation to determine which of the two choice dowels shares the same absolute extent as the target then children should perform below chance in the relative foil condition and above chance in the control condition. If children do not rely on the container as a standard but use some other strategy such as imposing an external standard on the target, performance should be identical in the two conditions. Note that both conditions use the same dowel stimuli permitting a direct comparison of performance between the two conditions. The only physical difference between the two conditions is whether the target and choice containers are the same size or are different sizes.

Participants

Twenty 4-year-old children (10 boys and 10 girls) participated in Experiment 1. The mean age of the children was 50 months (range = 42–54 months). Children were tested at the University of Chicago Laboratory School and were given stickers for their participation.

Materials

Two different sizes of glass containers (ungraduated cylinders without bases) were used: The shorter containers were 13 cm high and the taller containers were 18 cm high, both were 3.5 cm in diameter. The target stimuli were wooden dowels that varied in height; each had a diameter of 3.3 cm. The heights of the dowels consisted of five proportions of each container. For example, for Trial 3, one dowel had a height of 6.5 cm (one half the height of the small container) and the other had a height of 9 cm (one half the height of the large container). The five proportions of each container result in five distinct stimulus combinations that were used as target and foil stimuli in the trials in each condition. The heights of the stimulus pairs are provided in Table 1.

TABLE 1
Stimuli Dimensions for Experiments 1 and 2

<i>Stimulus Set</i>	<i>Smaller Dowel Stimuli</i>	<i>Larger Dowel Stimuli</i>
1	1.6	2.3
2	4.3	5.9
3	6.5	9.0
4	8.6	11.0
5 ^a	11.4	15.8

^aSee note in the design and procedure sections for Experiments 1 and 2 regarding counterbalancing for this trial.

Stimuli were presented on a small stage. The base of the stage measured 12 × 24 in. (30.5 × 61 cm) and stood 24 in. (61 cm) from the ground. The back of the stage was 20 in. (51 cm) high; black felt covered the back and base of the stage and was draped to diminish the salience of the edges.

Design and Procedure

Experimental trials consisted of two parts: an initial target presentation of a target stimuli inside a container and a choice task in which the child had to discriminate the target dowel from a foil dowel, both of which are presented inside containers. Each child was tested in the control condition and the relative foil condition, each condition consisted of 5 trials. In the control condition, the containers used in the initial target presentation and choice task were identical in height; in the relative foil condition, the containers used in the initial target presentation differed in height from the choice task containers by 5 cm; however the two choice task containers were identical to each other. This manipulation resulted in a stimulus set for which the foil dowel had the same relation to the changed container as the target had in the initial container. Each participant discriminated the same 5 stimuli combinations twice: once in the control condition and once in the relative foil condition. However, we counterbalanced which of the two dowels served as the target or the foil within each subject so that if a child saw the taller dowel as a target in the control condition, they would see the shorter dowel as a target in the relative foil condition. Likewise, the size of the containers were counterbalanced so that in half of the relative foil trials the taller container was used in the initial target presentation and the shorter container in the choice task and the change in size of the containers reversed on the remaining trials. One trial (Stimulus Set 5) could not be counterbalanced this way and was presented in the same manner for all children. For this stimulus set, the target stimulus was always the short dowel in the small container because counterbalancing the change in container size on this trial would result in a choice task in which the target dowel would be taller than the shorter

container. Trials were presented in two pseudorandomized orders where container height (short container in initial target presentation; tall container in discrimination task and vice versa) was yoked.

Participants were led into a quiet room adjoining their classrooms and were asked to hold a stuffed-animal dog. The experimenter stood behind the stage facing the child. Participants were told

We're going to play a little game with my dog Toby. Do you know how dogs like to hide bones? Well, Toby hides his bones underneath these toy blocks. Then he puts the block in one of my glasses. [Experimenter manipulates the animal so it appears to put a block in a container then shows the participant the bone through the bottom of the container.] Sometimes I like to trick Toby, so when he isn't looking I put his block in a different glass. [Experimenter places block from the original container to the novel container]. Now, your job is to help Toby find his bone. You need to remember the exact size of the block where he hid his bone. When I show you these two glasses [Experimenter presents the target block and a foil in the novel-sized containers]. I want you to point to the block where Toby hid his bone. Remember, you have to point to the block that is the same exact size as the one where Toby hid his bone.

Children were asked to repeat the story to verify its comprehension.

Two practice trials were given using stimulus sizes that were not used in the experiment. On each trial, the experimenter placed a dowel in a container on the stage for 7 sec and said, "This is the one where Toby hid his bone." The target was then removed and after a delay the target and foil were presented. The experimenter then said, "Now point to the block where Toby hid his bone." After the child pointed, the experimenter lifted the container to reveal the hidden bone. After these two trials the actual experiment began. Test trials were identical to the practice trials with the exception that the correct choice was not revealed.

Results and Discussion

A response was coded as correct if the child pointed to the dowel with the same extent as the target. An arcsin transformation was performed on proportion correct scores for each child in each condition. A split-plot analysis of variance (ANOVA) was performed with sex and order as between-subject factors and condition as a within-subjects factor. This analysis yielded a significant main effect of condition, $F(1, 16) = 53.95, p < .001$, with no other significant effects or interactions. The main effect of condition reflected a significant difference in performance between the relative foil and control conditions. However, the critical analysis is the comparison of performance to chance (.50). In the control

condition, the proportion (and standard error) of trials where children correctly chose the target was .82 (.038), which is significantly greater than chance, $t(19) = 9.2, p < .001$. In the relative foil condition, the proportion of correct responses was .34 (.043), which is significantly less than chance, $t(19) = -3.65, p < .01$.

If children did not use the initial relation between the target and container in the encoding phase of the test trial, performance in the choice task should have been identical in both conditions, as children discriminated between the same target and foil dowels. The significant difference in performance between conditions suggests that in both conditions, children used the initial relation between the target and container to identify the target from the foil in the choice task. However, although using the relation in the control condition resulted in correct responses in the discrimination task, using this initial relation in the relative foil condition resulted in incorrect responses in the discrimination task.

If the target's extent is encoded only relative to the container, and only the relation is used to determine the target from the foil, it follows that performance in the two conditions should be symmetric with respect to chance. However, we found that children correctly choose the target in the control condition more often than they incorrectly choose the foil in the relative foil condition. The difference between performance in the control condition (.82) and the additive complement of performance in the relative foil condition (.66) is significant, $t(19) = 3.43, p < .01$. One possible explanation for this finding is that children can encode extent without relying on the relation encoded in the initial display but are unable to inhibit choosing the relative foil in the discrimination task due to its perceptual similarity to the target in the initial display. Children may misunderstand the task and choose the relative foil even though the task explicitly asks children to choose the dowel that has the same extent. A second possibility is that the control condition is conceptually simpler than the relative foil condition. Because the correct choice has the same relation to the container as the target in the initial display in the control condition, choosing the target dowel does not involve the additional step of inhibiting the relation encoded in the initial portion of the test trial.

To address these possibilities and provide a stronger test of the hypothesis that children use only the relation between the initial target and container to later discriminate the target from a foil, Experiment 2 introduces the 'unrelated foil' condition in which using the relation established in the initial display would lead to chance performance in the discrimination task. If children encode information about the extent of the target without relying on the relation established from the initial display, they should exhibit above-chance performance at choosing the target in the unrelated foil condition. However, if children do rely on the relation to encode extent, children's performance should be at chance in this condition as neither the target nor the foil has the same relation to the new container as the relation between the target and container in the initial display.

EXPERIMENT 2

In Experiment 2, we tested 4- and 8-year-olds to determine whether children before and after the emergence of conventional measurement skills are equally influenced by the relative information provided by an available standard. Both age groups were tested with the relative foil condition introduced in Experiment 1 and the unrelated foil condition shown in Figure 1.

Participants

Forty children (20 boys and 20 girls) participated in Experiment 2. There were two age groups: twenty 4-year-olds ($M = 49$ months; range = 45–52) and twenty 8-year-olds ($M = 98$ months; range = 94–104). Children were tested at the University of Chicago Laboratory School and were given stickers for their participation.

Materials and Apparatus

The materials and apparatus were identical to Experiment 1.

Design and Procedure

Children were tested with the two within-subjects conditions (the relative foil and unrelated foil conditions), each consisting of five trials. The trials in the unrelated foil condition were created by using the same dowel stimuli as were used in the relative foil trials but using the smaller set of dowels in the larger containers and vice versa. This manipulation disrupted the relation between the foil and the changed containers so that neither the target nor the foil shared the same relation to the changed container as the target in the initial container. The procedure and cover story were identical to that used in the relative foil condition. In the unrelated foil condition, a modification of Stimulus Set 5 was necessary as the 15.75 cm dowel was taller than the shorter container. In this stimulus set, the foil was a 7 cm tall dowel that had the same difference in extent between the target and foil (4.4 cm) as the difference between the target and foil in the relative foil condition. This trial was always conducted with the taller container in the encoding portion and the shorter containers in the discrimination portion of the test trial.

Results and Discussion

For the ANOVA an arcsin transformation was performed on the proportion of correct scores for each child in each condition. A split plot ANOVA was performed with age, order, and sex as between-subject factors and condition (relative foil and

unrelated foil) as the within-subjects factor. There was a significant main effect of age $F(1, 32) = 61.49, p < .001$, with 8-years-olds significantly outperforming 4-year-olds. In addition, there was a significant interaction between age and condition, $F(1, 32) = 54.23, p < .001$, such that there was a difference in performance between the two conditions for the 4-year-olds, whereas there was no difference in performance between conditions for the 8-year-olds. There were no other significant effects or interactions.

As in Experiment 1, the critical analysis is children's performance compared with chance (.50). For the 4-year-olds, proportion of trials where the child chose the target in the relative foil condition was .33 (.04), which is significantly less than chance, $t(19) = -4.07, p < .001$. Performance in this condition therefore replicated the performance of children in the relative foil condition of Experiment 1. However, in the unrelated foil condition, the proportion of trials in which children chose the target was .56 (.05), which does not differ significantly from chance, $t(19) = 1.33, p > .15$. This demonstrates that when neither the target nor the foil share the same relation to the new container as the target and the original container, 4-year-old children do not discriminate the target from the foil.

In contrast with the 4-year-olds, 8-year-olds performed well above chance in both conditions; in the relative foil condition, performance at choosing the target was .82 (.04), $t(19) = 9.20, p < .001$, and for unrelated foil condition performance was .78 (.04), $t(19) = 8.06, p < .001$. Thus, 8-year-olds did not rely only on the relation between the target dowel and the container for discriminating the target from the foil in the discrimination task. Rather, these children may impose an external or mental standard on the target dowel. The 8-year-olds' performance on this task may be attributable to understanding the conventions and logic of measurement, as by the age of eight most children have been exposed to discrete unit measurement techniques in school or at home. In addition, their ability to ignore the misleading relative information may also be attributable to a more domain-general shift in cognitive style, an issue we return to in the discussion.

These findings address the issue of the asymmetric performance between the two conditions in Experiment 1. Because the unrelated foil does not maintain the same relation to the changed container as the initial target in the original container, there should be no bias toward choosing the foil stimulus based on its perceptual similarity to the target. Hence, if children could impose a mental or external standard on the target, they would not choose the foil in the unrelated foil condition but exhibit above-chance performance. The fact that the 4-year-olds' performance is at chance suggests that young children are not imposing an external or mental standard and that the relation between the initial target and container is crucial for successfully discriminating the target from the foil. The asymmetric performance observed in Experiment 1 may be related to the conceptual simplicity of the control condition compared to the conceptual difficulty of inhibiting the initial relation in the relative foil condition.

GENERAL DISCUSSION

Prior work has shown that young children can encode the extent of a continuous quantity earlier than they acquire conventional measurement skills. However, no study has directly addressed the mechanism by which young children encode extent. The two experiments reported here support the hypothesis that 4-year-old children rely on the relation between a target object and a perceptually available standard object to encode extent.

The early ability to encode an object's extent may resemble conventional measurement in that both require encoding extent relative to a standard object. Yet, there are important differences between relative encoding of extent and mature discrete-unit measurement. A critical aspect of the ability to use conventional measurement is an understanding that the size of the standard must remain constant across target comparisons. We found that children can encode the extent of a target when the standard's extent remains constant across encoding and discrimination portions of the test trial. Under such conditions, encoding the relation between a target object and a perceptually available standard is sufficient for discriminating one object from another that differs in extent. When the extent of the standard object is changed, children no longer successfully retain information about an object's extent. However, the errors that children make are not random: 4-year-olds chose the foil dowel with the wrong extent but the same relation to the new container as the target had in the initial display. This reliance on relative information also explains why 4-year-olds perform at chance when neither target nor foil has the same relation as the target in the initial display.

These findings suggest that young children do not encode information about an object's absolute extent, only the relation between an object's absolute extent and the absolute extent of an available standard. However, by the age of 8 years, children no longer rely solely on the relation provided by an available standard and can ignore misleading relative information provided by an available standard. It is possible that their understanding of measurement allows them to impose a mental or external standard on the initial target dowel.

Huttenlocher and colleagues (2002) found that the ability to determine extent in the absence of a standard begins to emerge around the age of four. However, the 4-year-olds in this study still only relied on the initial relation between the dowel and container. This discrepancy may reflect the fragile nature of the emerging ability to ignore relative information provided by an aligned standard object. In the absence of such an object, 4-year-olds may be able to encode extent by imposing a standard on the target. However, in the presence of an aligned standard, children may rely on an earlier strategy of automatically encoding extent relative to this standard. One possible explanation for this phenomenon is that younger children tend to be influenced by contextual information to a greater extent than older children or adults. Witkin (1962) demonstrated this phenomenon using a variety of

tests such as the rod-in-frame task that young children are field-dependent in cognitive style and that over the course of development become increasingly field-independent. If the containers are perceived as the target's context, it may be more difficult for children to ignore this relative information provided by the standard even though they have the cognitive capacity to impose an external standard on the target.

Adults also rely on the more primitive strategy of encoding extent relative to an available standard but only under perceptually impoverished conditions. Rock and Ebenholtz (1957) demonstrated that adults are influenced by misleading relative information in estimating the absolute extent of a line in an experimental task where participants saw a 3 in. target line in a 9 in. rectangular frame. The participant reproduced the absolute length of the line in a 27 in. frame. The task was conducted with luminous lines and frames in a darkened room to control for the influence of other possible external cues. Rock and Ebenholtz (1959) found that the mean response was around 6 in., indicating that the relation between the target line and frame influenced participants' judgment of the absolute extent of the line.

The finding of this research may help to explain early success on tasks in which infants and children encode the extent of continuous quantities such as length, distance, or volume. In these studies it is generally the case that the target object was aligned with a second object that could be used to establish the relative extent between the object and part of the stimulus display. In addition, the presence of such standards may explain the discrepancy between the Piaget and colleagues (1960) claimed that measurement emerges in the school age years and later findings of infant sensitivity to extent. The Piaget and colleagues (1960) finding of late emerging sensitivity to extent occurred in a condition in which a standard was neither present nor aligned with the target but required the child to physically impose the standard on the target object.

The results of this study may also have implications for studies in other domains of quantification. Most studies that explore infant and young children's understanding of discrete number do not control for the fact that changes in discrete number covaries with changes in continuous amount (Clearfield & Mix, 1999). It is possible that infants and young children rely on a continuous variable such as area to encode changes in amount on tasks that test discrete number knowledge. Changing the discrete number of objects on an array also changes the relation between the amount of 'object' and the amount of 'array' in the display, and it may be this change in relation, rather than a change in number, that infants and young children use to discriminate changes in discrete number. Future work on early number knowledge should control for this confound by carefully controlling for the relation between the objects and the array across changes in discrete number.

Learning to impose a standard that is neither perceptually present nor aligned may be the conceptual challenge for the development of conventional measurement techniques (Nunes & Bryant, 1996). Encoding extent relative to an available,

aligned standard may be an ontogenetically primitive skill that leads to the mature ability to measure. The conceptual challenge for the emergence of measurement skills may originate in a shift in strategy from relying on standards obtained from the immediate perceptual environment to imposing external standards (such as rulers) on a target extent.

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